Smart Systems for Urban Water Demand Management

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joint work with **A.K. Sampathirao**, **P. Patrinos** & **A. Bemporad**.

IMT Institute for Advanced Studies Lucca

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Today's talk

We will learn how to:

- **model** water networks
- **identify** control objectives
- **make decisions** under uncertainty
- **formulate** MPC problems
- **devise** algorithms to solve them
- **parallelise** them on GPUs


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Literature overview
Control
Control-oriented models
Forecasting
The MPC concept
Worst-case MPC
Stochastic MPC
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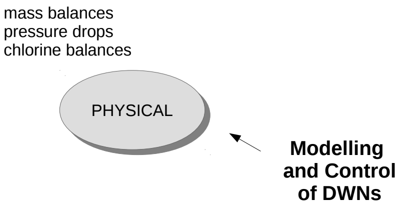
Proximal operator
Convex conjugate
Duality
APG algorithm
DWN SMPC problems
Parallelisation
Simulations
KPIs
Simulation results
Conclusions

Modoller's todos



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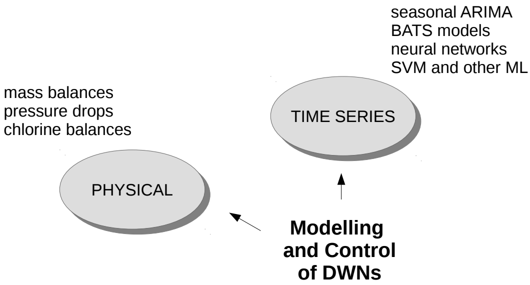

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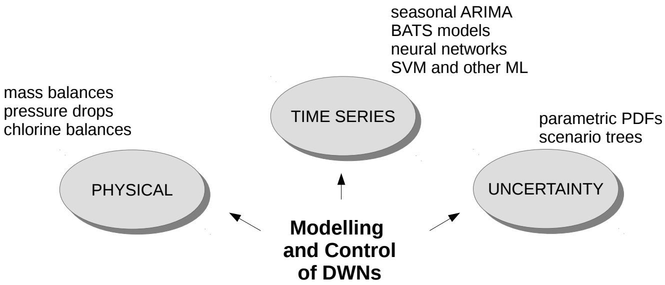

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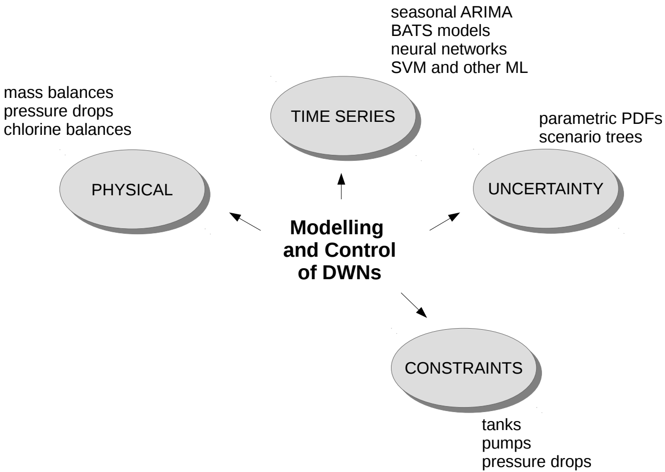

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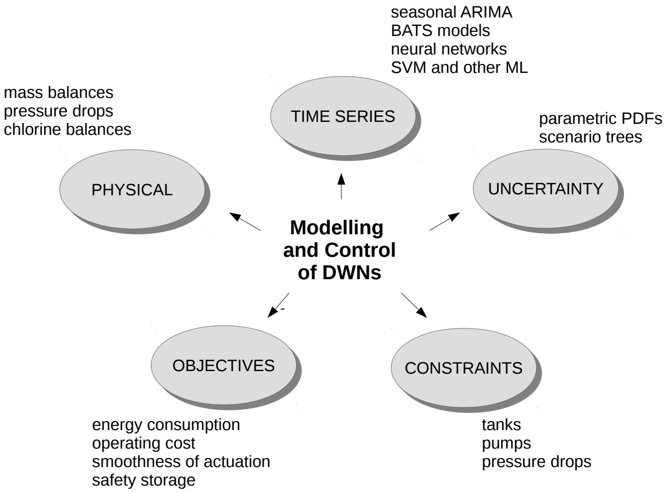
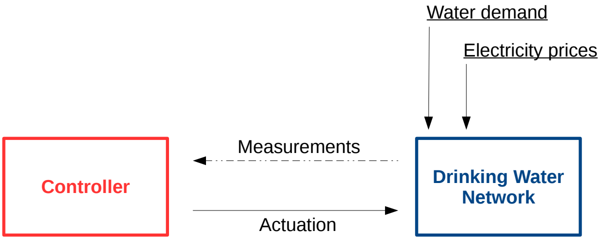
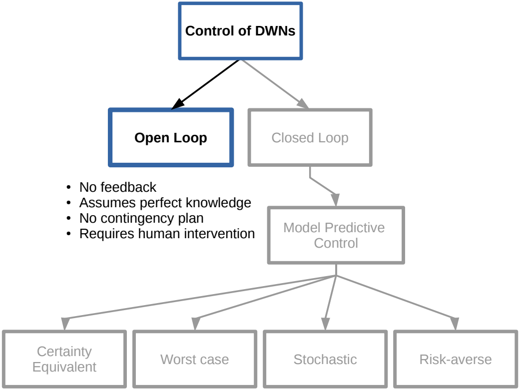


Control of water networks



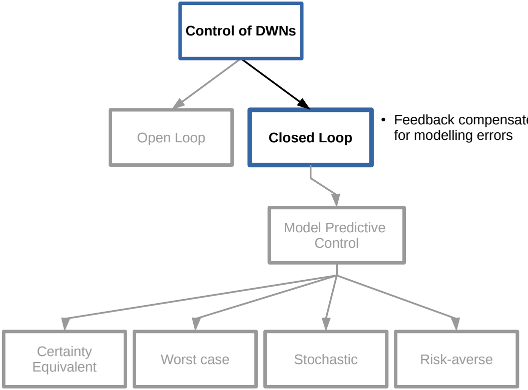
Taxonomy of control methodologies



- No feedback
- Assumes perfect knowledge
- No contingency plan
- Requires human intervention

Creasy 1998; Yu *et al.*, 1994; Zessler and Shamir, 1989

Taxonomy of control methodologies



- Feedback compensates for modelling errors

Creasy 1998; Yu *et al.*, 1994; Zessler and Shamir, 1989

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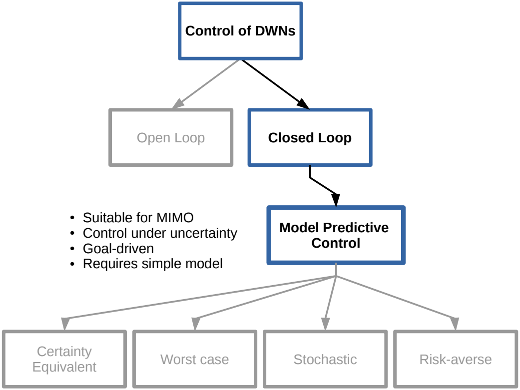

Creasy 1998; Yu *et al.*, 1994; Zessler and Shamir, 1989

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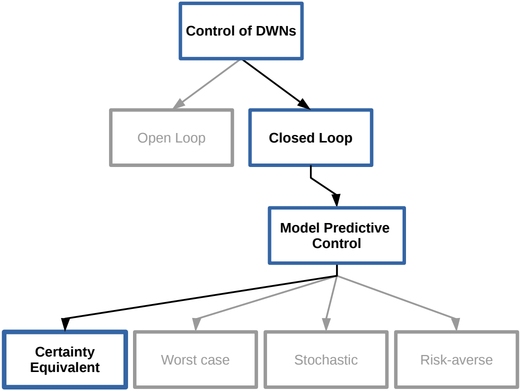
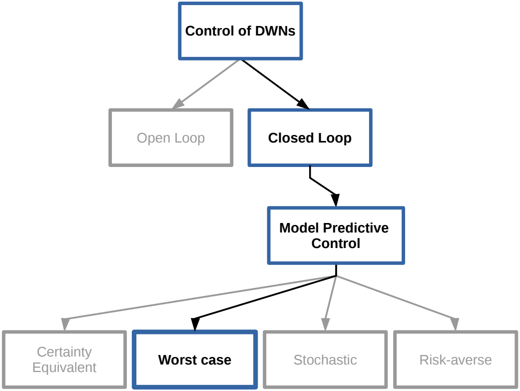


- Model assumed accurate
- Constraints may be violated
- Suboptimal (we can do better)

Sampathirao *et al.*, 2014; Bakker *et al.*, 2013; Leirens *et al.*, 2010; Ocampo *et al.*, 2009.

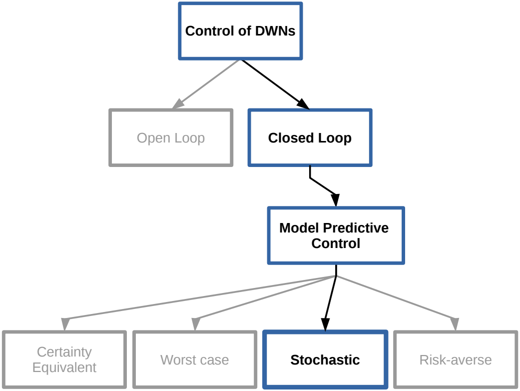
Taxonomy of control methodologies



- Pessimistic and conservative
- Leads to small domain of attraction

Ocampo *et al.*, 2009 and 2010.

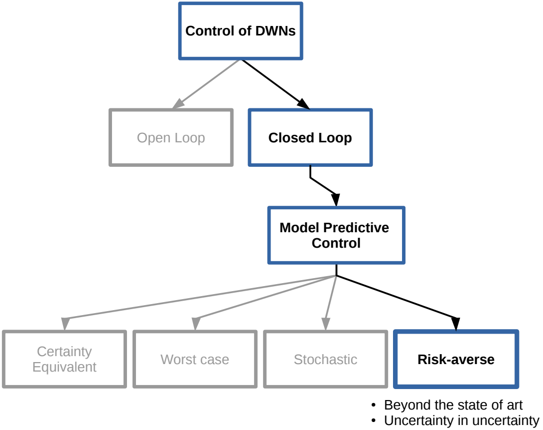
Taxonomy of control methodologies



- Makes use of probabilistic info
- Less conservative, more realistic

Sampathirao *et al.*, 2014; Goryashko and Nemirovski, 2014; Tran and Brdys, 2009, Watkins and McKinney, 1997

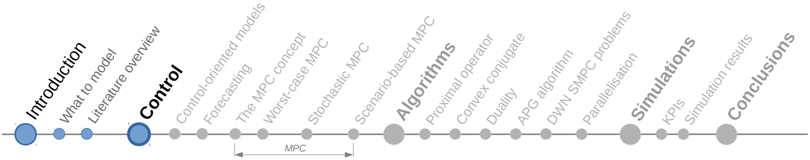
Taxonomy of control methodologies



Sampathirao *et al.*, 2016

Objectives

To **MODEL** (water demands, hydraulics, uncertainty, etc), pose a stochastic predictive **CONTROL** problem (define objectives, constraints) and devise algorithms to **SOLVE** it numerically.



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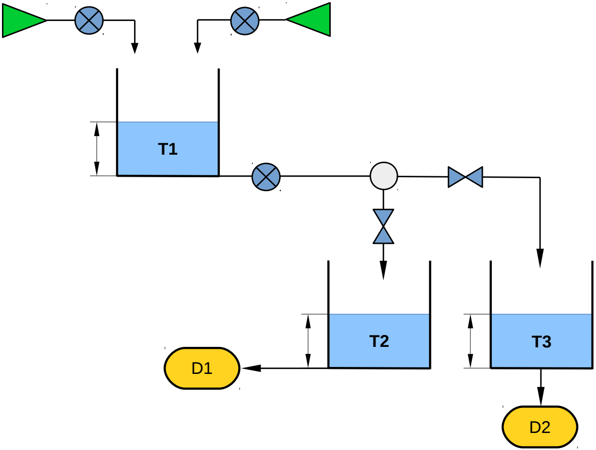
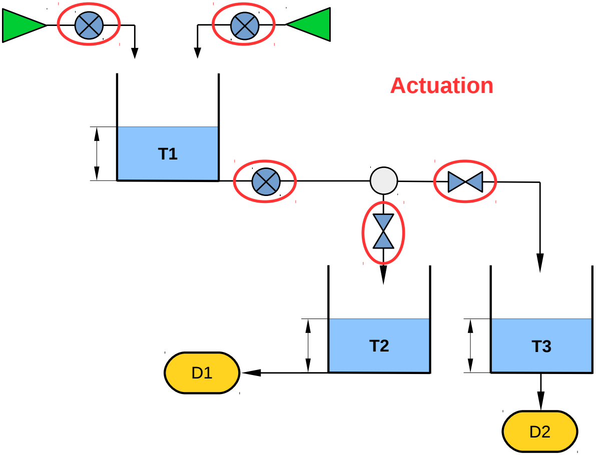
Control-oriented models



Control-oriented models



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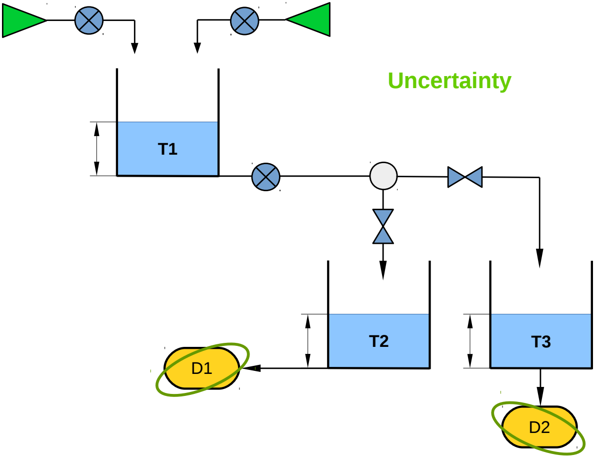
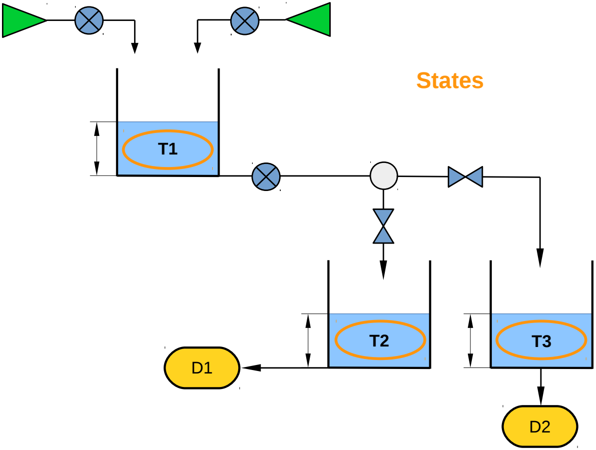
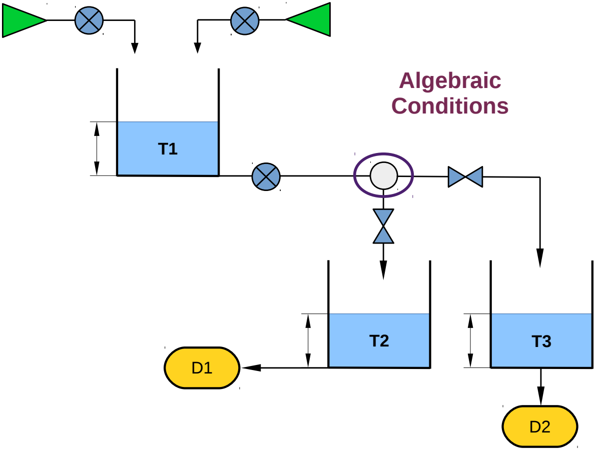


Control-oriented models

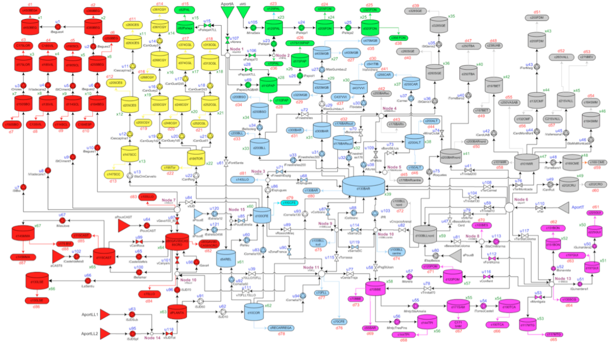


States

Control-oriented models



Our case study



DWN of Barcelona: 63 tanks, 114 pumping stations and valves, 88 demand nodes & 17 pipe intersection nodes.

Control-oriented models

Simple mass balance equation (in discrete time)

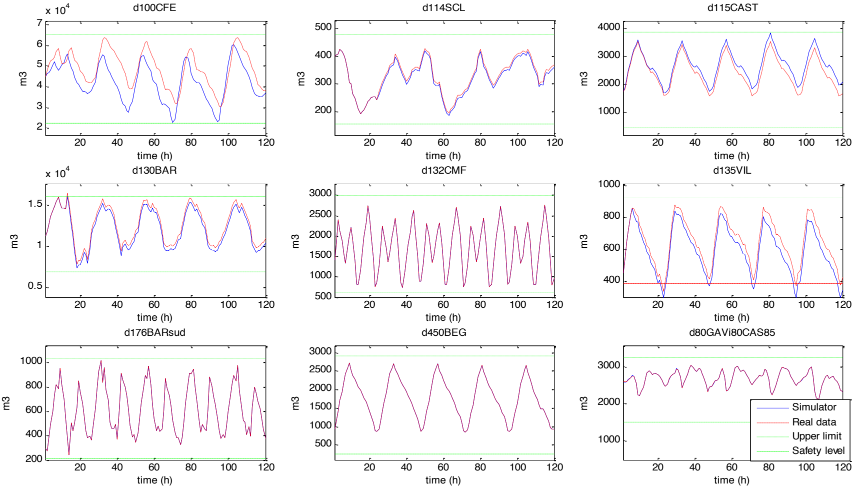
$$x_{k+1} = Ax_k + Bu_k + G_d d_k,$$
$$0 = Eu_k + E_d d_k,$$

x_k: tank volumes, u_k: flows (controlled by pumping), d_k: demands — along with the constraints

$$x_{\min} \leq x_k \leq x_{\max},$$
$$u_{\min} \leq u_k \leq u_{\max}.$$

Sampathirao, Sopasakis *et al.*, 2016, 2014; Wang *et al.*, 2014; Ocampo *et al.*, 2010; Ocampo *et al.*, 2009.

Control-oriented models



Source: EFFINET, deliverable report D2.1

Demand forecasting

Demand prediction concept:

$$d_{k+j}(\epsilon_j) = \hat{d}_{k+j|k} + \epsilon_j$$

where

1. d_{k+j}: actual demand at time $k+j$
2. $\hat{d}_{k+j|k}$: prediction of d_{k+j} using info up to time k
3. ϵ_j: j-step-ahead prediction error

and $\hat{d}_{k+j|k}$ is a function of observable quantities up to time k.

Sampathirao, Sopasakis *et al.*, 2016.

Demand forecasting

Common approaches:

1. Neglect the error: $(\epsilon_0, \epsilon_1, \ldots, \epsilon_N) \approxeq (0, 0, \ldots, 0)$

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3. Independent normal distributions: $\epsilon_j \sim \mathcal{N}(m_j, \sigma_j^2)$

Demand forecasting

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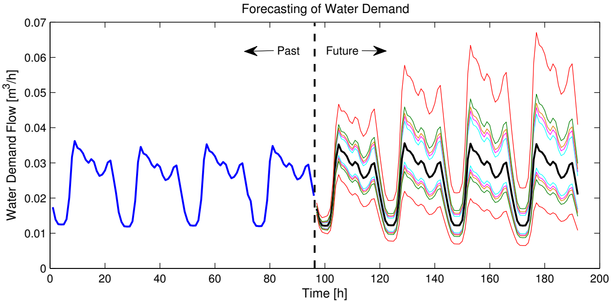
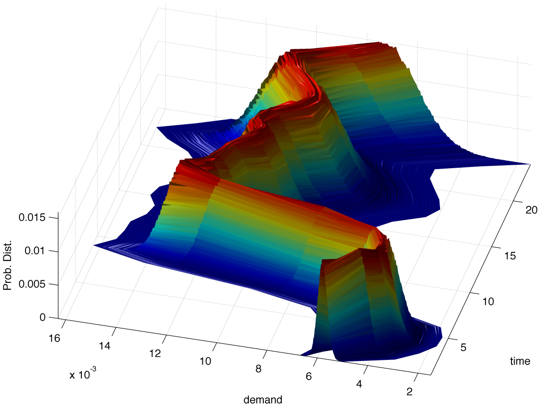
3. Independent normal distributions: $\epsilon_j \sim \mathcal{N}(m_j, \sigma_j^2)$
4. $(\epsilon_0, \epsilon_1, \ldots, \epsilon_N)$ is random and admits finitely many values

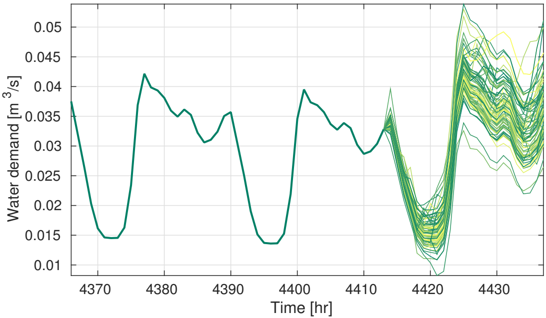
Error bounds



Continuous independent errors



Predicted scenarios



Control objectives

Stage costs:

1. Economic cost: $\ell^w(u_k, k) = W_\alpha(\alpha_1 + \alpha_{2,k})' u_k$

We define $\Delta u_k = u_k - u_{k-1}$

Sampathirao *et al.*, 2014; Cong Cong *et al.*, 2014

Control objectives

Stage costs:

1. Economic cost: $\ell^w(u_k, k) = W_\alpha(\alpha_1 + \alpha_{2,k})' u_k$
2. Smooth operation cost: $\ell^\Delta(\Delta u_k) = \Delta u_k' W_u \Delta u_k$

We define $\Delta u_k = u_k - u_{k-1}$

Sampathirao *et al.*, 2014; Cong Cong *et al.*, 2014

Control objectives

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2. Smooth operation cost: $\ell^\Delta(\Delta u_k) = \Delta u_k' W_u \Delta u_k$
3. Safe operation cost: $\ell^S(x_k) = W_x \|[x_s - x_k]_+\|$

We define $\Delta u_k = u_k - u_{k-1}$

Sampathirao *et al.*, 2014; Cong Cong *et al.*, 2014

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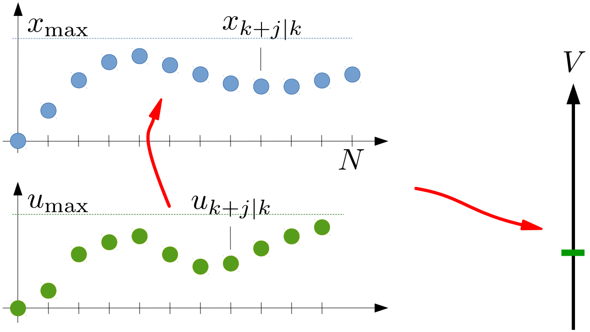
3. Safe operation cost: $\ell^S(x_k) = W_x \| [x_s - x_k]_+ \|$
4. Total cost: $\ell = \ell^w + \ell^\Delta + \ell^S$.

We define $\Delta u_k = u_k - u_{k-1}$

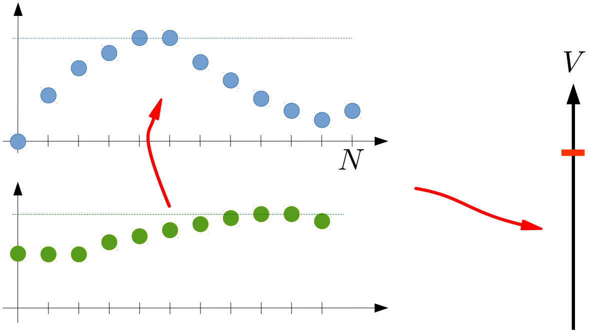
Sampathirao *et al.*, 2014; Cong Cong *et al.*, 2014

Model Predictive Control



Rawlings and Mayne, 2009

Model Predictive Control



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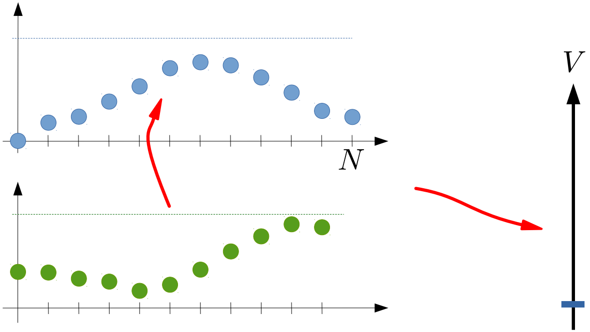

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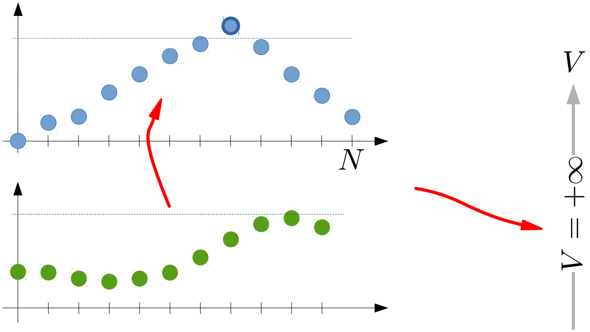

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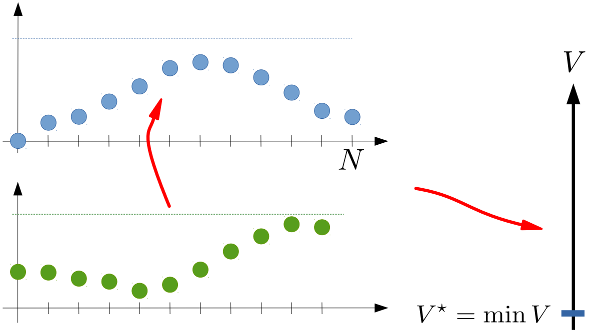
Model Predictive Control



Rawlings and Mayne, 2009

Model Predictive Control

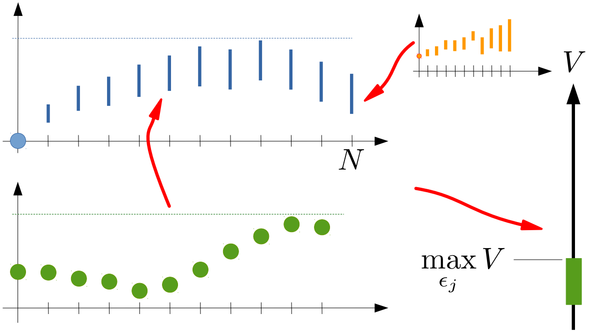
Problem formulation

$$\underset{\pi=(\{u_{k+j|k}\}_j,\{x_{k+j|k}\}_j)}{\text{minimise}} \; V(\pi) := \sum_{j=0}^{N-1} \ell(x_{k+j|k}, u_{k+j|k}, u_{k+j-1|k}, k),$$

subject to

$x_{k+j+1\|k} = Ax_{k+j\|k} + Bu_{k+j\|k} + G_d\hat{d}_{k+j\|k}$	dynamics
$Eu_{k+j\|k} + E_d\hat{d}_{k+j\|k} = 0$	algebraic
$x_{\min} \leq x_{k+j\|k} \leq x_{\max}$	vol. constr.
$u_{\min} \leq u_{k+j\|k} \leq u_{\max}$	flow constr.
$x_{k\|k} = x_k,\; u_{k-1\|k} = u_{k-1}$	initial cond.

Worst-case MPC



Rawlings and Mayne, 2009

Worst-case MPC

Problem formulation

$$\underset{\pi=(\{u_{k+j|k}\}_j,\{x_{k+j|k}\}_j)}{\text{minimise}} \; \max_{d_{k+j|k}} V(\pi),$$

subject to

$d_{k+j} = \hat{d}_{k+j\|k} + \epsilon_j$	predictions
$\epsilon_j \in \mathcal{E}_j$	err. bounds
$x_{k+j+1\|k} = Ax_{k+j\|k} + Bu_{k+j\|k} + G_d d_{k+j}$	dynamics
$Eu_{k+j\|k} + E_d d_{k+j} = 0$	algebraic
$x_{\min} \leq x_{k+j\|k} \leq x_{\max}$	vol. constr.
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$x_{k\|k} = x_k,\; u_{k-1\|k} = u_{k-1}$	initial cond.

Here $u_{k+j|k}$ is a function of ϵ_j – not a fixed value!

Worst-case MPC

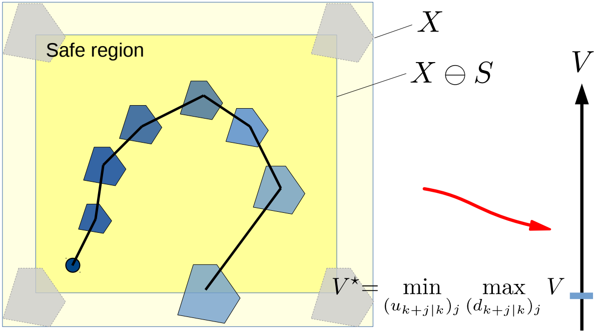
Attention! We are looking for control laws (functions) $u_{k+j|k}$. We may parametrise (why?) these functions as

$$u_{k+j|k} = K_j e_j + b_j,$$

and solve for K_j and b_j.

There exist other parametrisations as well.

Worst-case MPC (tube-based)



Rawlings and Mayne, 2009

Worst-case MPC (tube-based)

Problem formulation

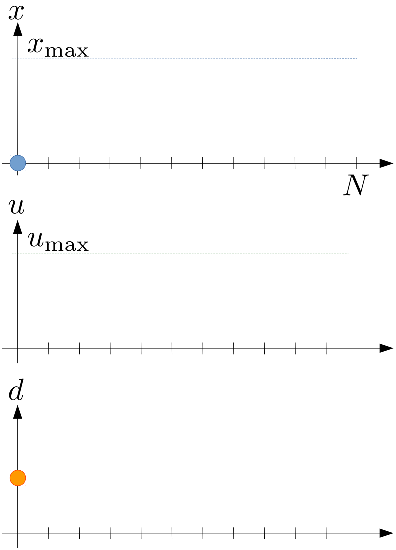
$$\underset{\pi=(\{u_{k+j|k}\}_j,\{x_{k+j|k}\}_j)}{\text{minimise}} \quad V(\pi),$$

subject to

$x_{k+j+1\|k} = Ax_{k+j\|k} + Bu_{k+j\|k} + G_d\hat{d}_{k+j\|k}$	dynamics
$Eu_{k+j\|k} + E_d\hat{d}_{k+j\|k} = 0$	algebraic
$x_{k+j\|k} \in X \ominus S$	volume constr.
$u_{\min} \leq u_{k+j\|k} \leq u_{\max}$	flow constr.
$x_{k\|k} = x_k,\ u_{k-1\|k} = u_{k-1}$	initial cond.

* the constraint $Eu_{k+j|k} + E_d d_{k+j|k} = 0$ (certainty-equivalent) will not be satisfied for all ϵ_j

Stochastic MPC



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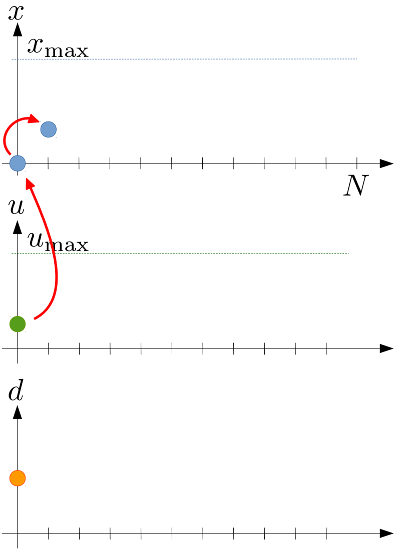

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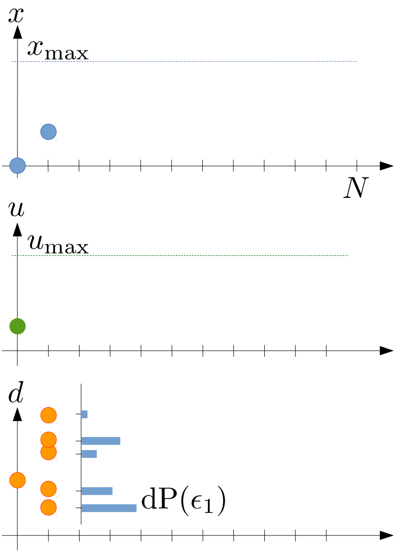

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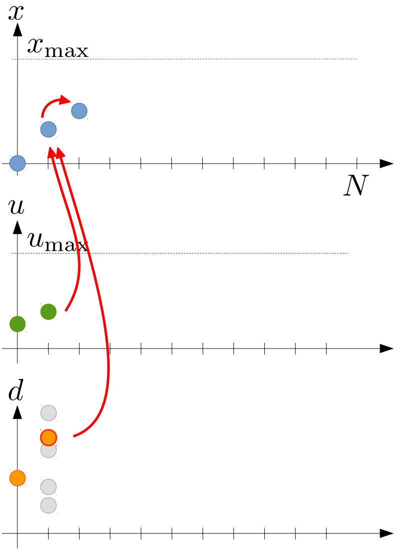

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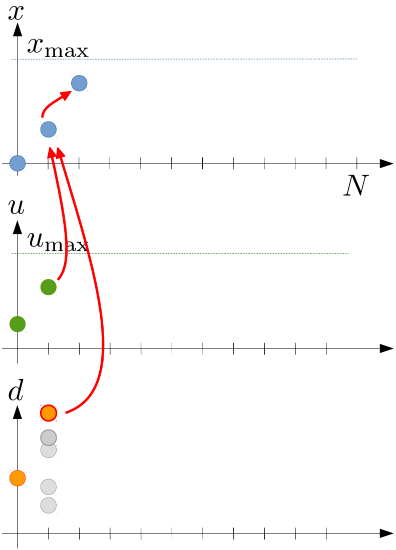

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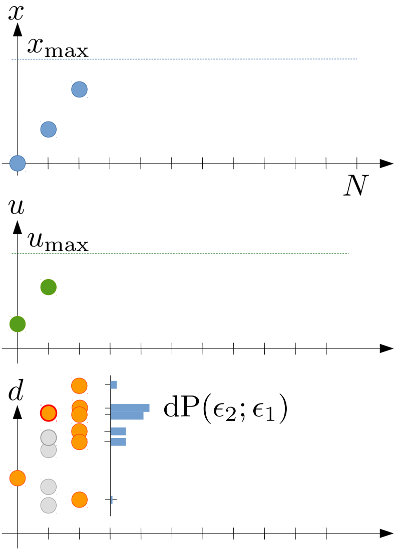

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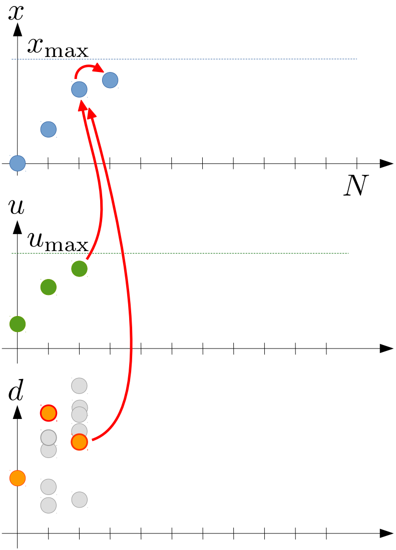

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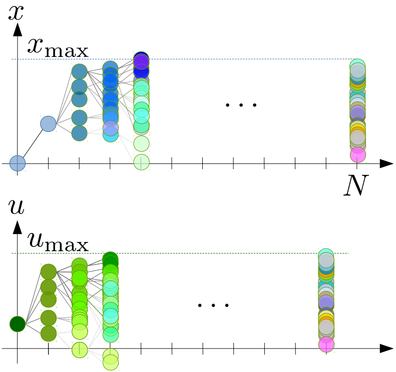

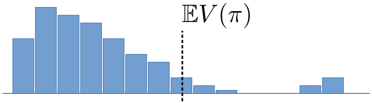

Stochastic MPC

Problem formulation

$$\underset{\pi=(\{u_{k+j|k}\}_j,\{x_{k+j|k}\}_j)}{\text{minimise}} \quad \mathbb{E}V(\pi),$$

subject to

$x_{k+j+1\|k} = Ax_{k+j\|k} + Bu_{k+j\|k} + G_d\hat{d}_{k+j\|k}(\epsilon_j)$	dynamics
$Eu_{k+j\|k} + E_d d_{k+j}(\epsilon_j) = 0$	alg. cond.
$x_{\min} \leq x_{k+j\|k} \leq x_{\max}$	vol. constr.
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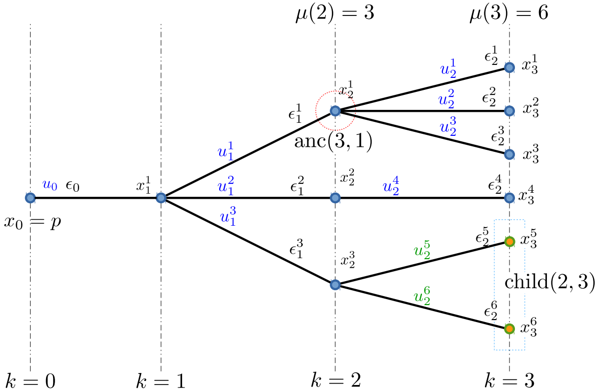
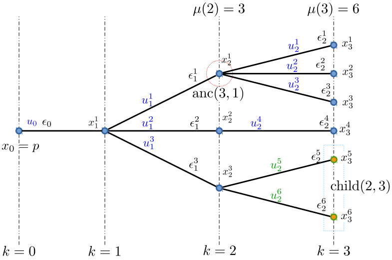
$x_{k\|k} = x_k,\ u_{k-1\|k} = u_{k-1}$	initial cond.

again, we're looking for control laws $u_{k+j|k}(\epsilon_j)$.

Scenario trees



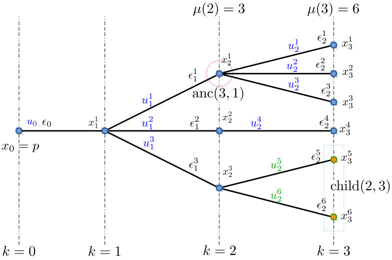
Scenario trees



Demand forecasting:

$$d^i_{k+j} = \hat{d}_{k+j|k} + \epsilon^i_j$$

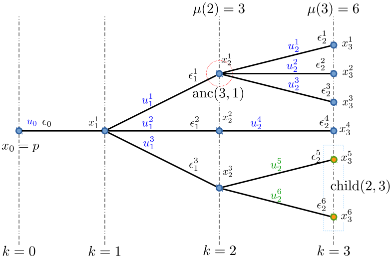
Scenario trees



Input-disturbance coupling:

$$E u^{i}_{k+j|k} + E_d d^{i}_{k+j} = 0$$

Scenario trees



System dynamics:

$$x^i_{k+j+1|k} = f(x^{\mathrm{anc}(j+1,i)}_{k+j|k}, u^i_{k+j|k}, d^i_{k+j|k})$$

Scenario-based stochastic MPC

Problem formulation:

$$\operatorname*{minimise}_{\pi=(\{u_{k+j^i|k}\}_{i,j},\{x_{k+j^i|k}\}_{i,j})} \overbrace{\sum_{j=0}^{N-1}\sum_{i=1}^{\mu(j)} p_j^i \ell_j^i}^{\mathbb{E}V(\pi)},$$

subject to

$$x^i_{k+j+1|k} = f(x^{\mathrm{anc}(j+1,i)}_{k+j|k}, u^i_{k+j|k}, d^i_{k+j|k}) \quad \text{dynamics}$$

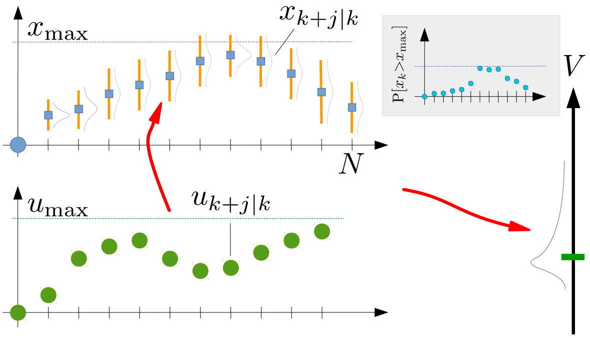
$$Eu^i_{k+j|k} + E_d \hat{d}^i_{k+j|k} = 0 \quad \text{algebraic}$$

$$x_{\min} \leq x^i_{k+j|k} \leq x_{\max} \quad \text{volume constr.}$$

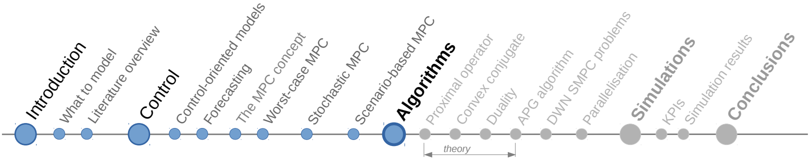
$$u_{\min} \leq u^i_{k+j|k} \leq u_{\max} \quad \text{flow constr.}$$

$$x^1_{k|k} = x_k,\ u^1_{k-1|k} = u_{k-1} \quad \text{initial cond.}$$

Stochastic MPC (chance constraints)



Grosso *et al.*, 2014.



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The proximal operator

Let $g : \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$ be a *proper*, *closed* function and $\gamma > 0$. Define

$$\operatorname{prox}_{\gamma g}(v) = \arg\min_z \left\{ g(z) + \tfrac{1}{2\gamma} \|z - v\|^2 \right\}$$

If this is **easy to compute** we call g **prox-friendly**.

Parikh and Boyd, 2014; Combettes and Pesquet, 2010.

The proximal operator

Example 1. Take

$$g(x) = \delta(x \mid C) = \begin{cases} 0, & \text{if } x \in C \\ +\infty, & \text{otherwise} \end{cases}$$

Then, $\operatorname{prox}_{\gamma g}(v) = \operatorname{proj}(v \mid C)$.

Parikh and Boyd, 2014; Combettes and Pesquet, 2010.

The proximal operator

Example 2. Take

$$g(x) = d(x \mid C) = \inf_{y \in C} \|y - x\|$$

Then,

$$\mathrm{prox}_{\lambda g}(v) = \begin{cases} v + \frac{\mathrm{proj}(v|C) - v}{d(v|\mathcal{C})}, & \text{if } d(v \mid \mathcal{C}) > \lambda \\ \mathrm{proj}(v \mid C), & \text{otherwise} \end{cases}$$

Parikh and Boyd, 2014; Combettes and Pesquet, 2010.

The proximal operator

Key Property. Suppose g is given as

$$g(x) = \sum_{i=1}^{\kappa} g_i(x_i),$$

Parikh and Boyd, 2014; Combettes and Pesquet, 2010.

The proximal operator

Key Property. Suppose g is given as

$$g(x) = \sum_{i=1}^{\kappa} g_i(x_i),$$

then

$$(\mathrm{prox}_{\lambda g}(v))_i = \mathrm{prox}_{\lambda g_i}(v_i).$$

Parikh and Boyd, 2014; Combettes and Pesquet, 2010.

Convex Conjugate

Let $f : \mathbb{R} \to \mathbb{R} \cup \{+\infty\}$ be convex, proper and closed. We define its **convex conjugate** as

$$f^*(y) = \sup_x \left\{ x'y - f(x) \right\}.$$

If f is strongly convex, then f^* is continuously diff/ble (Rockaffelar and Wets, 2009; Prop. 12.60).

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$$f^*(y) = \sup_x \left\{ x'y - f(x) \right\}.$$

When f^* is differentiable, then

$$\nabla f^*(y) = \arg\min_z \{x'y + f(z)\}.$$

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Convex Conjugate

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$$f^*(y) = \sup_x \left\{x'y - f(x)\right\}.$$

When f^* is differentiable, then

$$\nabla f^*(y) = \arg\min_z \{x'y + f(z)\}.$$

If f is prox-friendly, then

$$\mathrm{prox}_{\lambda f}(v) + \lambda \mathrm{prox}_{\lambda^{-1} f^*}(\lambda^{-1} v) = v.$$

If f is strongly convex, then f^* is continuously diff/ble (Rockaffelar and Wets, 2009; Prop. 12.60).

Forward-Backward Splitting

The **forward-backward splitting** is the representation of an optimisation problem as follows

$$\operatorname*{minimise}_{z}\ f(z) + g(z),$$

where

- f, g: closed, convex
- f: diff/ble with Lipschitz gradient
- g: prox-friendly

FB Splittings are not unique.

Forward-Backward Splitting

Example 1. ℓ_1-regularized least squares:

$$\underset{z}{\text{minimise}}\ \frac{1}{2}\|Az - b\|^2 + \|z\|_1.$$

Forward-Backward Splitting

Example 2. Box-constrained QP

$$\operatorname*{minimise}_z \tfrac{1}{2}z'Qz + q'z + \delta(z \mid C),$$

where $C = \{z : z_{\min} \leq z \leq z_{\max}\}$.

Forward-Backward Splitting

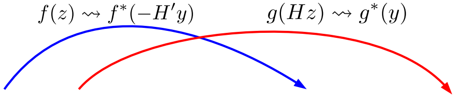
Example 3. Constrained QP

$$\operatorname*{minimise}_z \ \tfrac{1}{2}z'Qz + q'z + \delta(Hz \mid C).$$

but, $g(z) := \delta(Hz \mid C)$ is not prox-friendly!

Dual optimisation problem

If $g(z)$ is prox-friendly, but $g(Hz)$ is not we may formulate the **dual** optimisation problem

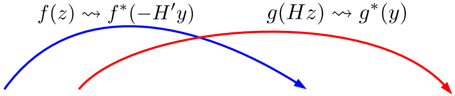


$$\text{minimise } f(z) + g(Hz) \qquad \text{minimise } f^*(-H'y) + g^*(y)$$

Fenchel duality generalises Lagrangian duality (Rockafellar, 1972).

Dual optimisation problem

If $g(z)$ is prox-friendly, but $g(Hz)$ is not we may formulate the **dual** optimisation problem



$$\text{minimise } f(z) + g(Hz) \qquad \text{minimise } f^*(-H'y) + g^*(y)$$

Under certain conditions these two problems have the same minimum and

$$z^\star = \nabla f^*(-H'y^\star).$$

Fenchel duality generalises Lagrangian duality (Rockafellar, 1972).

Proximal gradient algorithm

The **proximal gradient** method for solving

$$\operatorname*{minimise}_{z} \; f(z) + g(z)$$

where f is differentiable with L-Lipschitz gradient runs

$$z^{\nu+1} = \operatorname{prox}_{\gamma g}(z^{\nu} - \gamma \nabla f(z^{\nu})),$$

with $\gamma \in (0, L^{-1})$.

Dual proximal gradient algorithm

The **proximal gradient** method applied to the dual

$$\operatorname*{minimise}_{z}\ f^*(-H'y) + g^*(y)$$

where f^* is differentiable with L-Lipschitz gradient is

$$y^{\nu+1} = \operatorname{prox}_{\gamma g^*}(y^\nu + \gamma H \nabla f^*(-H'y^\nu))$$

If f is L^{-1}-strongly convex, then f^* has L-Lipschitz gradient.

Dual proximal gradient algorithm

The dual proximal gradient method

$$y^{\nu+1} = \mathrm{prox}_{\gamma g^*}(y^\nu + \gamma H \nabla f^*(-H' y^\nu))$$

can be written as

$$\begin{aligned} z^\nu &= \nabla f^*(-H' y^\nu) \\ t^\nu &= \mathrm{prox}_{\lambda^{-1} g}(\lambda^{-1} y^\nu + H z^\nu) \\ y^{\nu+1} &= y^\nu + \lambda(H z^\nu - t^\nu). \end{aligned}$$

Dual proximal gradient algorithm

Nesterov's **accelerated** proximal gradient method converges as $\mathcal{O}(1/k^2)$ instead of $\mathcal{O}(1/k)$:

$$
\begin{aligned}
w^\nu &= y^\nu + \theta_\nu(\theta_{\nu-1}^{-1} - 1)(y^\nu - y^{\nu-1}) \\
z^\nu &= \nabla f^*(-H' w^\nu) \\
t^\nu &= \mathrm{prox}_{\lambda^{-1} g}(\lambda^{-1} w^\nu + H z^\nu) \\
y^{\nu+1} &= w^\nu + \lambda(H z^\nu - t^\nu) \\
\theta_{\nu+1} &= \tfrac{1}{2}\left(\sqrt{\theta_\nu^4 + 4\theta_\nu^2} - \theta_\nu^2\right)
\end{aligned}
$$

with $\theta_0 = \theta_{-1} = 1$ and $y_0 = y_{-1} = 0$ (Nesterov, 1983).

The DWN control problem

$$\mathbb{P}: \underset{\pi=(\{u_{k+j^i|k}\}_{i,j},\{x_{k+j^i|k}\}_{i,j})}{\text{minimise}} \overbrace{\sum_{j=0}^{N-1}\sum_{i=1}^{\mu(j)} p_j^i \ell_j^i}^{\mathbb{E}V(\pi)},$$

subject to

$$x^i_{k+j+1|k} = f(x^{\text{anc}(j+1,i)}_{k+j|k}, u^i_{k+j|k}, d^i_{k+j|k}) \qquad \text{dynamics}$$

$$Eu^i_{k+j|k} + E_d \hat{d}^i_{k+j|k} = 0 \qquad \text{algebraic}$$

$$x_{\min} \leq x^i_{k+j|k} \leq x_{\max} \qquad \text{volume constr.}$$

$$u_{\min} \leq u^i_{k+j|k} \leq u_{\max} \qquad \text{flow constr.}$$

$$x^1_{k|k} = x_k,\ u^1_{k-1|k} = u_{k-1} \qquad \text{initial cond.}$$

We will write this problem as: $\text{minimise}_z\, f(z) + g(Hz)$.

The DWN control problem

Let $z = \{x_j^i, u_j^i\}$ and define

$$f(z) = \sum_{j=0}^{N-1} \sum_{i=1}^{\mu(j)} p_j^i(\ell^w(u_j^i) + \ell^\Delta(\Delta u_j^i)) + \delta(u_j^i | \Phi_1(d_j^i)) \\ + \delta(x_{j+1}^i, u_j^i, x_j^{\mathrm{anc}(j+1,i)} | \Phi_2(d_j^i)),$$

where

$$\Phi_1(d) = \{u : Eu + E_d d = 0\}$$

and

$$\Phi_2(d) = \{(x^+, x, u) : x^+ = Ax + Bu + G_d d\}$$

Sampathirao *et al.*, 2016.

The DWN control problem

In other words:

$$f(z) = \text{smooth cost} + \text{dynamics} + \text{alg equations}$$

and

$$\begin{aligned} g(z) &= \text{all the rest} \\ &= \text{nonsmooth cost} + \text{constraints} \\ &= \sum_{j=0}^{N-1} \sum_{i=1}^{\mu(j)} \ell^S(x_j^i) + \delta(x_j^i \mid X) + \delta(u_j^i \mid U), \end{aligned}$$

but this g is **not prox-friendly** (it is not separable!).

Sampathirao *et al.*, 2016.

The DWN control problem

We create a **copy** of $\{x_j^i\}_{j,i}$ which we denote by χ_j^i and introduce

$$t = (\{x_j^i\}_{j,i}, \{\chi_j^i\}_{j,i}, \{u_j^i\}_{j,i})$$

with $x_j^i = \chi_j^i$. Then

$$g(t) = \sum_{j=0}^{N-1} \sum_{i=1}^{\mu(j)} \ell^S(x_j^i) + \delta(\chi_j^i \mid X) + \delta(u_j^i \mid U)$$

is **prox-friendly**.

It is $t = Hz$.

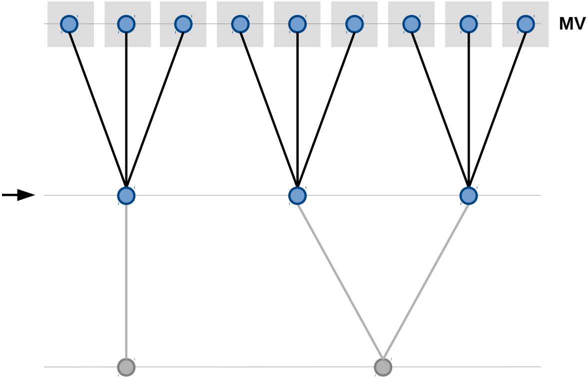
Dual gradient computation

To compute the dual gradient we use

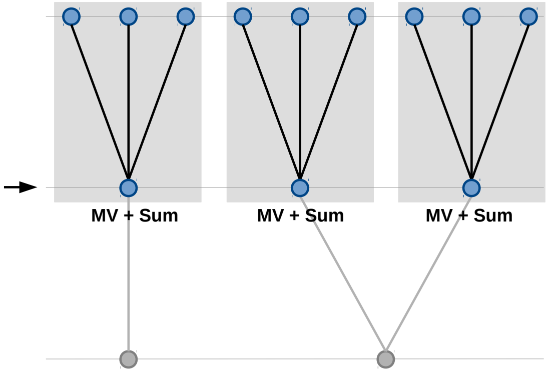
$$\begin{aligned}\nabla f^*(y) &= \arg\min_z \{x'y + f(z)\} \\ &= \underset{\substack{z:\text{dynamics} \\ Eu_j^i + E_d d_j^i = 0}}{\arg\min} \{x'y + \sum_{j,i} \text{quadratic}(z_j^i)\}\end{aligned}$$

This is an **equality-constrained** quadratic problem which can be solved very efficiently using **dynamic programming**.

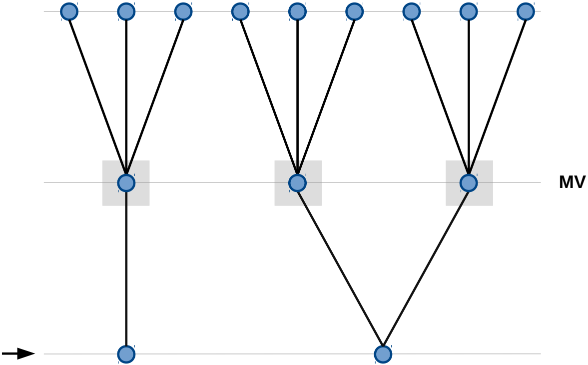
Dual gradient computation



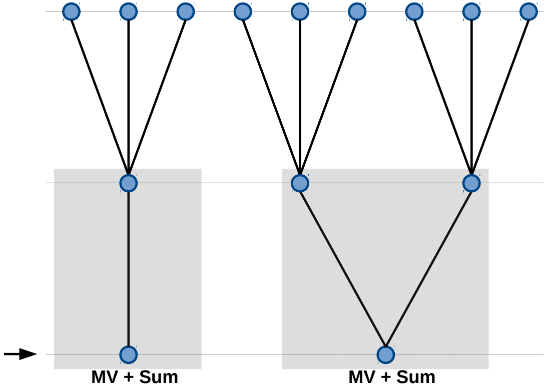
Dual gradient computation

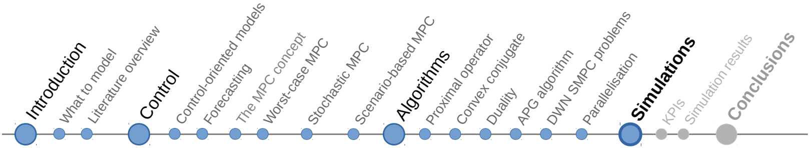


Dual gradient computation



Dual gradient computation

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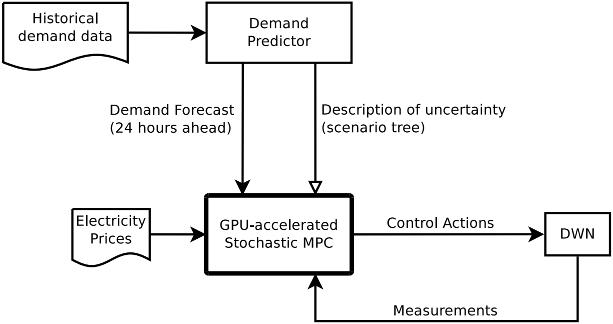
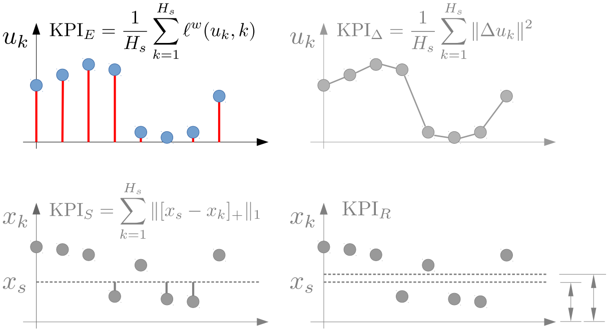
Control scheme

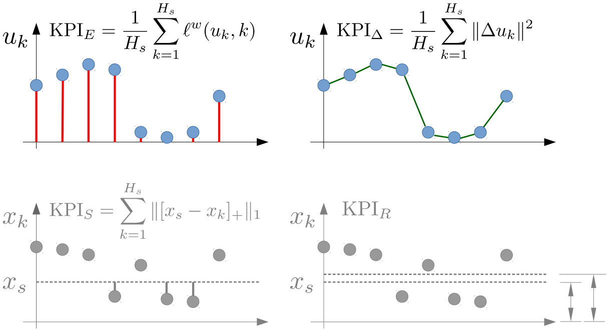


KPIs



u_k
$\mathrm{KPI}_E = \frac{1}{H_s}\sum_{k=1}^{H_s} \ell^w(u_k, k)$
u_k
$\mathrm{KPI}_\Delta = \frac{1}{H_s}\sum_{k=1}^{H_s} \|\Delta u_k\|^2$
x_k
$\mathrm{KPI}_S = \sum_{k=1}^{H_s} \|[x_s - x_k]_+\|_1$
x_s
x_k
KPI_R
x_s

KPIs



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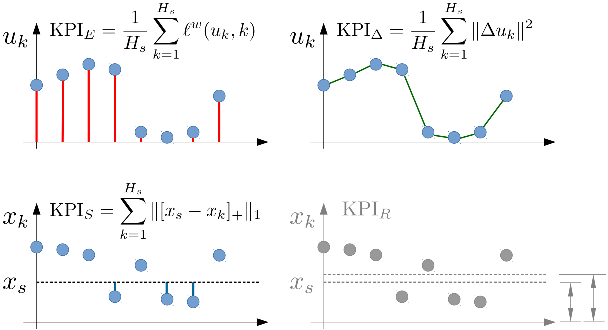
x_s
x_k
KPI_R
x_s

KPIs



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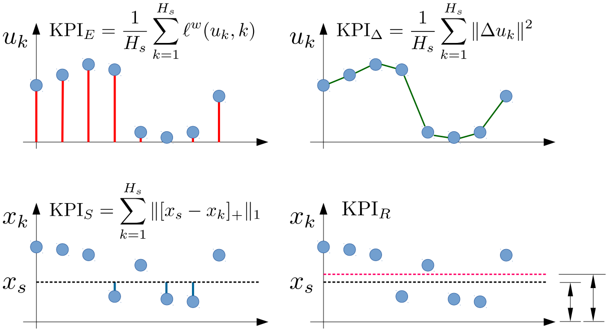
KPI_R
x_s

KPIs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x_k
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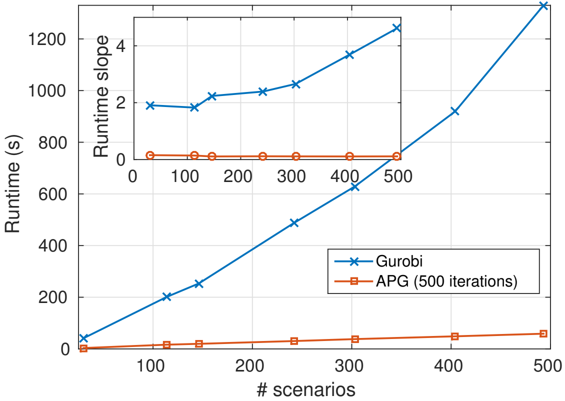
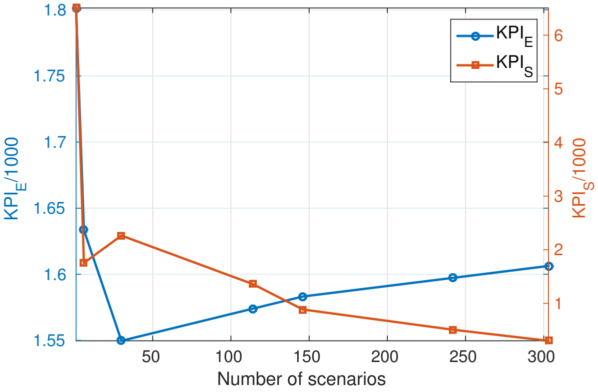
x_s
x_k
KPI_R
x_s

it is fast

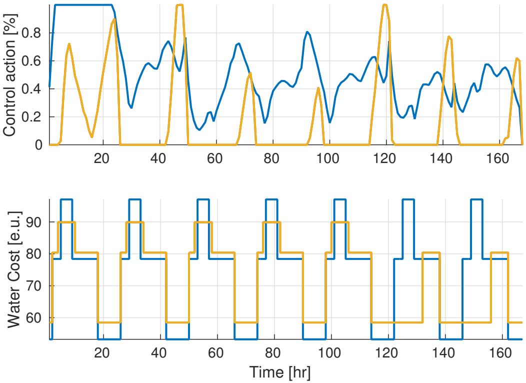


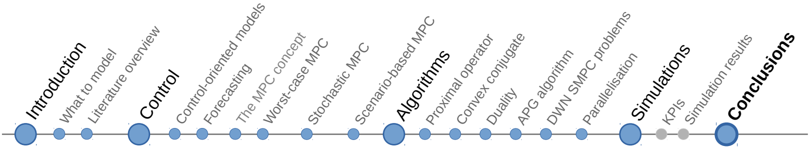
Runtime (s)
1200
1000
800
600
400
200
0
100
200
300
400
500
scenarios
Runtime slope
4
2
0
0
100
200
300
400
500
Gurobi
APG (500 iterations)

it is efficient



Closed-loop simulations





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Open problems

- **Nonlinear**

Problem: introduce nonlinear pressure drop equations

Open problems

- **Nonlinear**
- **Risk-averse**

Challenge: the distribution of future errors is not exactly known

Open problems

- **Nonlinear**
- **Risk-averse**
- **Distributed**

Problems: spatial decomposition, communication constraints

Open problems

- **Nonlinear**
- **Risk-averse**
- **Distributed**
- **Robust Economic MPC**

Questions: performance guarantees, recursive feasibility

Open problems

- **Nonlinear**
- **Risk-averse**
- **Distributed**
- **Robust Economic MPC**
- **Faster Algorithms**

Ongoing work: quasi-Newtonian LBFGS-type algorithms

Thank you for your attention!

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